

2.2 PROBABILISTIC PILOT-BEHAVIOR MODELS FOR CLEAR-AIR TURBULENCE AVOIDANCE MANEUVERS

Robert Sharman*
National Center for Atmospheric Research, Boulder, CO

Jimmy Krozel, and Victor Klimenko
Metron Aviation, Inc., Dulles, VA

1. INTRODUCTION

In order to understand how cruise-level turbulence impacts the National Airspace System (NAS), an analysis was provided for how pilots tactically respond when encountering Clear-Air Turbulence (CAT). Given probabilistic estimates of severe CAT from the Graphical Turbulence Guidance (GTG) forecast model (Sharman et al., 2006) and aircraft trajectory data describing potential turbulence encounters in the NAS, models are developed to estimate relationships between the type and magnitude of maneuvers versus the existence of turbulence in the upcoming sector of airspace (a sector-based approach) or along the upcoming trajectory (a trajectory-based approach). Results indicate that pilot responses to CAT depend on several factors including user class, weight class, physical class, aircraft type, as well as airline policies. This model is a starting point for increasing the capacity of the NAS while maintaining aviation safety within the Next Generation Air Transportation System (NextGen). Only after well-formulated pilot-behavior models are built can we then pursue integration of such models into the decision support tools that pilots, dispatchers, and controllers will use in NextGen to make intelligent decisions to plan safe and efficient air travel in the NAS.

2. BACKGROUND

At cruising altitudes of commercial aviation, there are three common sources of turbulence: 1) turbulence associated with convective clouds (Convective-Induced Turbulence or CIT); 2) turbulence associated with enhanced wind shears and reduced stabilities in the vicinity of jet streams, the tropopause, and upper-level fronts, commonly referred to as CAT since it often occurs in clear air or sometimes in stratiform clouds; and 3) turbulence associated with the breaking of gravity waves above mountainous terrain, which also often occurs in clear air and is termed Mountain Wave Turbulence (MWT) (Lester, 1993).

Traditionally, Pilot Reports (PIREPs) have been used to describe the bumpiness in flight caused by turbulent eddies. PIREPs describe the turbulence as “light,” “moderate,” “severe,” or “extreme,” but this is subjective and aircraft-dependent. Yet, for large commercial aircraft predominantly in cruise flight, PIREP intensities are remarkably consistent between various aircraft types (Wolff and Sharman, 2008).

An aircraft-independent turbulence intensity scale, by international agreement (International Civil Aviation Organization, 2007) is described quantitatively by the energy dissipation rate (edr), ϵ , or actually $\epsilon^{1/3}$ [$\text{m}^{2/3}\text{s}^{-1}$]. For large aircraft, the edr values roughly translate to PIREP intensity as follows (Federal Aviation Regulations/Aeronautical Information Manual, 2008): light corresponds to edrs in the range 0.125–0.375 $\text{m}^{2/3}\text{s}^{-1}$, moderate to edrs of 0.375–0.625 $\text{m}^{2/3}\text{s}^{-1}$, severe to edrs in the range 0.625–0.875 $\text{m}^{2/3}\text{s}^{-1}$, and extreme to edr values greater than 0.625 $\text{m}^{2/3}\text{s}^{-1}$.

For turbulence forecasting, edr is the preferred output. However, since direct edr observations are currently available on only a few United Airlines (UAL) and Delta Air Lines (DAL) aircraft, PIREPs are still used in the turbulence forecast tuning and verification process (Sharman et al., 2006).

For aviation, moderate and severe turbulence are the key drivers of pilot maneuvers. Ride comfort and safety are not usually affected by light turbulence, however, higher turbulence intensities, viz., Moderate-or-Greater (MoG) or Severe-or-Greater (SoG) turbulence, often trigger pilot maneuvers. When encountering moderate turbulence, passengers and crew suffer annoyance and discomfort, and occasionally personal injury if individuals and equipment are not secured; and in severe encounters, the turbulence may produce aircraft structural damage.

As shown in Figure 1, climatologies of turbulence indicate where in the NAS turbulence is likely to occur. These climatologies are based on MoG and SoG PIREPs using the techniques in (Wolff and Sharman, 2008). As shown, severe turbulence encounters seem to be concentrated in three major areas over the US: the first over the eastern seaboard, a second over the Florida peninsula, and a third over the Rockies and Sierra Nevada mountain ranges in the West. This distribution is consistent with the MoG PIREPs distribution shown in (Wolff and Sharman, 2008), who attributed the maximum over the eastern seaboard to primarily jet stream-related CAT, while the maxima over Florida and to some extent the southeast are associated with the increased frequency of convective storms (CIT), especially in summer. The largest continuous area of severe turbulence is over the Colorado Rockies, and is due primarily to MWT.

Climatologies are important considerations for strategic air traffic route planning, because the maxima in the Northeast impact particularly dense traffic areas, while the MWT over the center of the country imposes long-lasting hazards that affect transcontinental flights.

* Corresponding author address: Dr. Robert Sharman, National Center for Atmospheric Research, Research Applications Laboratory, P.O. Box 3000, Boulder, CO 80307; email: sharman@ucar.edu.

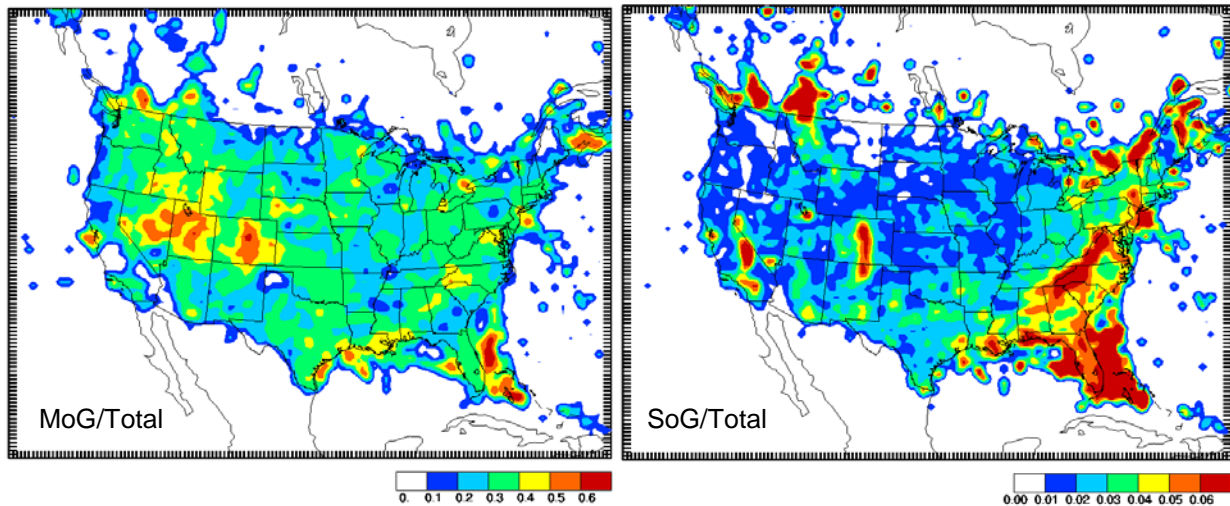


Figure 1: Relative MoG, SoG climatologies for the NAS based on 0 – FL550 and 15 years of PIREPs (1993-2007).

3. PILOT RESPONSE TO TURBULENCE

From a pilot's perspective, moderate turbulence may require changes in altitude, attitude, or indicated air speed as pilots seek Flight Levels (FLs) clear of turbulence. MoG turbulence can essentially close en route airspace flight levels given that passenger comfort and safety are a high priority for many airlines.

SoG turbulence causes large, abrupt changes in flight profiles, which may cause momentary loss of aircraft control. Forecast or reported SoG turbulence is an immediate safety hazard which closes airspace and, if encountered, may require an aircraft diversion.

Pilots adopt avoidance actions depending on the type of turbulence expected. Tactical maneuvering to avoid CAT is usually accomplished through climbs and descents rather than lateral maneuvers, because CAT patches are typically much thinner in the vertical extent (median patch thickness is about 500 m) than in the horizontal extent (median horizontal dimension is about 50 km) (Vinnichenko, et al., 1980). CIT, associated with thunderstorms, is mostly circumvented as part of the tactical avoidance of the associated convective weather cells, typically sensed by airborne weather radar. CIT is also avoided when airline dispatchers strategically file flight plans that avoid forecasted weather systems.

Both the time scale and longevity of turbulence events also influence pilot maneuvers. Whereas the time scale of CIT events is a few minutes, CAT and MWT events are typically much longer-lived, with a median lifetime of approximately 6 hours (Vinnichenko, et al., 1980), and some MWT events last as long as 2 days. Thus, CAT avoidance can be planned out to several hours, allowing for lateral reroutes to be performed given a turbulence forecast.

For high-altitude sectors, MoG turbulence encounters are about equally divided between clear-air and in-cloud occurrences, although many in-cloud reports are actually in stratiform clouds associated with mid-latitude winter storms (Wolff and Sharman, 2008).

4. DATA SOURCES USED IN ANALYSIS

The analysis reported in this paper is based on flight track data from the FAA's Traffic Flow Management System (TFMS) and turbulence data from the Graphical Turbulence Guidance (GTG) system.

TFMS data was the source of flight track and flight plan data. This data source provided the flight call sign (indicating the name of the airline), time stamp (to the second), aircraft latitude and longitude (in minutes), and altitude (in hundreds of feet), all recorded at one-minute intervals. TFMS data also included aircraft physical class (jet, turboprop, piston.), user class (commercial, air taxi, General Aviation (GA), military, etc.), weight class (heavy, large, or small), as well as aircraft type (A319, B737, MD80, etc.).

GTG forecasts were the source of turbulence assessments. The GTG (Sharman et al., 2006) uses gridded output from NWP model forecasts to derive 12 turbulence diagnostics that are normalized to a combined intensity scale and then combined as a weighted sum with the relative weights computed to give the best agreement with the most recent available turbulence observations (i.e., PIREPs). The set of diagnostics is selected to ensure that the indices appropriately represent the variety of atmospheric processes that may be contributing to the existing turbulence conditions – e.g., Richardson number, Ellrod Index, frontogenesis function, vertical wind speed, etc. These indices are the most useful in forecasting upper-level CAT and MWT but are not as skillful for CIT. (This is one reason why our analysis did not focus on CIT avoidance maneuvers.) The resulting GTG combination is output in a gridded format corresponding to the input NWP Rapid Update Cycle (RUC) (Benjamin et al., 2004) horizontal grid structure (roughly 20 km grid spacing) and at 1000 ft increments from FL180 to FL460.

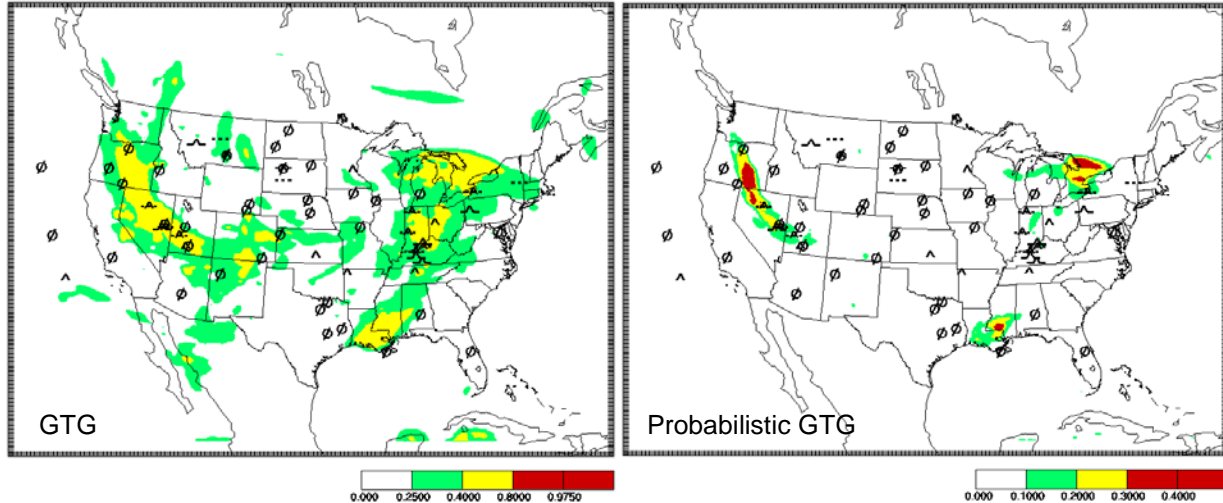


Figure 2: A GTG forecast for light, moderate, and severe CAT and the probabilistic GTG forecast for SoG CAT for FL360 on 09/22/2006 (color scales indicate two different quantities of information and are not directly comparable).

Probabilistic hourly GTG maps (zero lead time) were used in the present analysis. Probabilistic GTG maps (Figure 2) were derived by computing the percentage of the 12 turbulence diagnostics (indices) that agree on turbulence intensity category (we studied both moderate and severe) on a 20 km RUC grid-point-by-grid-point basis. Here, each of the 12 computed diagnostics is considered an equally likely representation of the 3D turbulence pattern at the analysis time. This approach is similar to that used in ensemble NWP forecasts, where different NWP models and model configurations are used to develop a measure of uncertainty associated with the forecasts (e.g., Kalnay, 2003).

The percentage agreement between the indices is termed a “probability”, however, this quantity is not calibrated to true probabilities of encountering turbulence along a flight track of a given length – this would require information about the spatially and seasonally dependent probability density function of atmospheric turbulence, which is not well known. Thus it is merely a normalized value between 0 (no diagnostics indicating turbulence) and 1 (all diagnostics indicating turbulence), with higher values in this range

corresponding to higher likelihoods of encountering turbulence of a given intensity category (moderate or severe). For example, at a grid point where the GTG probabilistic value was 0.5, 6 of 12 diagnostics indicated turbulence. As shown in Figure 2 (right) for the probabilistic GTG map, there is a five-color key identified in the lower right, contoured with an increment of 0.1: white (no diagnostics indicated severe turbulence), green (0.1 or roughly one diagnostic indicated severe turbulence), yellow (0.2 or roughly two diagnostics indicated severe turbulence), and red (0.3 or roughly three-to-four diagnostics indicated severe turbulence), and orange (0.4 or roughly five or more diagnostics indicated severe turbulence).

5. AIRCRAFT TRAJECTORY ANALYSIS

Our trajectory analysis approach was (a) to use the probabilistic SoG GTG maps to estimate the probability of a severe turbulence encounter at a given Sector-Flight Level Pair (SFLP) or along a given trajectory-flight level pair (Figure 3), and (b) to build a quantitative relationship between the observed pilot behavior in this SFLP or trajectory-flight level pair and the GTG output.

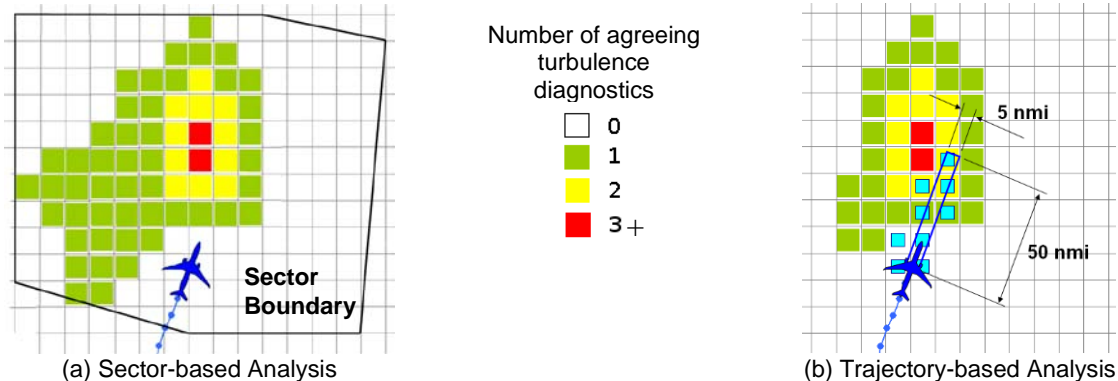


Figure 3: The sector-based and trajectory-based analysis approaches.

Table 1. Statistics for three days studied with significant turbulence activity.

Day	Number of MoG	95th Percentile MoG	Number of SoG	95th Percentile SoG
Oct. 28, 2006	281	248 for Year 2006	50	12 for Year 2006
Jan. 24, 2007	334	304 for Year 2007	27	20 for Year 2007
Oct. 23, 2007	327	304 for Year 2007	60	20 for Year 2007

In the sector-based approach, we express three probabilities of encountering severe turbulence in a given SFLP. The probability is expressed as the percentage of the SFLP (quantified in terms of the number of grid cells inside or touching the sector boundaries) covered by intensity equal to or greater than a specified probabilistic GTG threshold. For example, Figure 3(a) depicts a fictitious SFLP for a SoG GTG map. Grid cells in the sector are colored green (1 GTG diagnostic indicating SoG), yellow (2 GTG diagnostics indicating SoG), or red (3+ GTG diagnostics indicating SoG). In this example, the percentage of the sector that is green, yellow, or red is 34%, the percentage of the sector that is yellow or red is 7%, and the percentage of the sector that is red is 1%. These turbulence coverage percentages were then used for the basis of our sector-based analysis.

For the trajectory-based approach, we investigate the grid cells immediately ahead of the aircraft's trajectory: (a) For a given aircraft position, we calculate the projected direction of flight and identify the grid cells that overlap with a 5x50 nmi rectangle elongated in the projected direction of flight (light blue cells in Figure 3(b)). (b) For each of these cells, we determine the number of GTG indices that predict SoG turbulence (1/12 if one index indicates SoG, 2/12 if two indices indicate SoG, 3/12 if three indices indicate SoG etc.), and then average those values over all cells inside the rectangle. This gives us an average probabilistic GTG value for turbulent activity in the aircraft's immediate flight path. The latest hourly probabilistic GTG forecast is used for this purpose. For example, if the current time is from 1500 to 1600 UTC, the analysis of turbulence response is based on the 1500 UTC GTG forecast. (c) The 50 nmi stretch of the flight track starting from the current position of the aircraft from TFMS data was examined to identify if a pilot initiated a maneuver.

Only aircraft that were initially in straight and level en route flight prior to entering the forecast turbulence were analyzed. Aircraft that were initially climbing or descending were not considered since these aircraft are often in a transition phase of flight traveling to or from airports, with passengers buckled up in preparation for landing or takeoff, and therefore typically fly through moderate turbulence with minimal safety risk.

Pilot response to turbulence was classified into the following categories:

- No Response – aircraft remains in straight and level flight.
- Climbed – aircraft climbed two or more FL.
- Descended – aircraft descended two or more FL.
- Re-routed – aircraft re-routed around the sector where turbulence was present, based on the filed flight plan that passed through the sector of interest.

Additionally, we recorded airline name, aircraft type, user class, physical class, and weight class.

Statistics were computed for the type, magnitude, and frequency of the pilots' turbulence-avoidance maneuvers – and the ensuing changes in air traffic density within a sector boundary – as a function of the number of GTG indices indicating SoG.

For the sector-based analysis, over 8,200 flights flying through more than 300 sectors were analyzed. We studied two severe CAT events that occurred on Oct. 28, 2006 in central eastern US, and another on Jan. 24, 2007 over the Colorado Rockies. Data collected from these events showed no cloud-ground lightning strikes, indicating these days had CAT with no evidence of CIT. For both days, we limited our scope to the 1500–1900 UTC time window, and the 24,000 ft–45,000 ft altitude range in the first case and the 30,000 ft–45,000 ft band in the second case.

For the trajectory-based analysis, over 61,000 flights were analyzed, resulting in over 108,000 aircraft-turbulence encounters. We studied three days (Table 1): Oct. 28, 2006, Jan. 24, 2007, and Oct. 23, 2007. In all cases, we limited our scope to 1500–1900 UTC and the 20,000 ft–46,000 ft altitude range. On these three days, GTG displayed elevated probabilities of SoG turbulence over large areas of the NAS with minimal or no convective activity in the vicinity of the SoG events. This allowed us to ascribe the observed turbulence-avoidance behavior specifically to the presence of CAT or MWT.

Turbulence-avoidance statistics were computed for 18 individual groups of aircraft in an attempt to identify various factors that affect pilots' decisions when reacting to or anticipating turbulence:

- | | |
|-------------------------------------|-------------------------------------|
| 1. All aircraft | 10. Airline B heavy commercial jets |
| 2. All heavy commercial jets | 11. Airline C heavy commercial jets |
| 3. All heavy cargo jets | 12. Airline A large commercial jets |
| 4. All large commercial jets | 13. Airline B large commercial jets |
| 5. All large air taxi jets | 14. Airline C large commercial jets |
| 6. All large GA jets | 15. Airline D large commercial jets |
| 7. All small GA jets | 16. Airline E large commercial jets |
| 8. All large commercial turboprops | 17. Airline F large commercial jets |
| 9. All large air taxi turboprops | |
| 10. Airline A heavy commercial jets | |

For turboprops (#8 and #9), insufficient statistics were accumulated to present meaningful results. Airlines A, B, C, D, E, and F are six of the top 10 US air carriers.

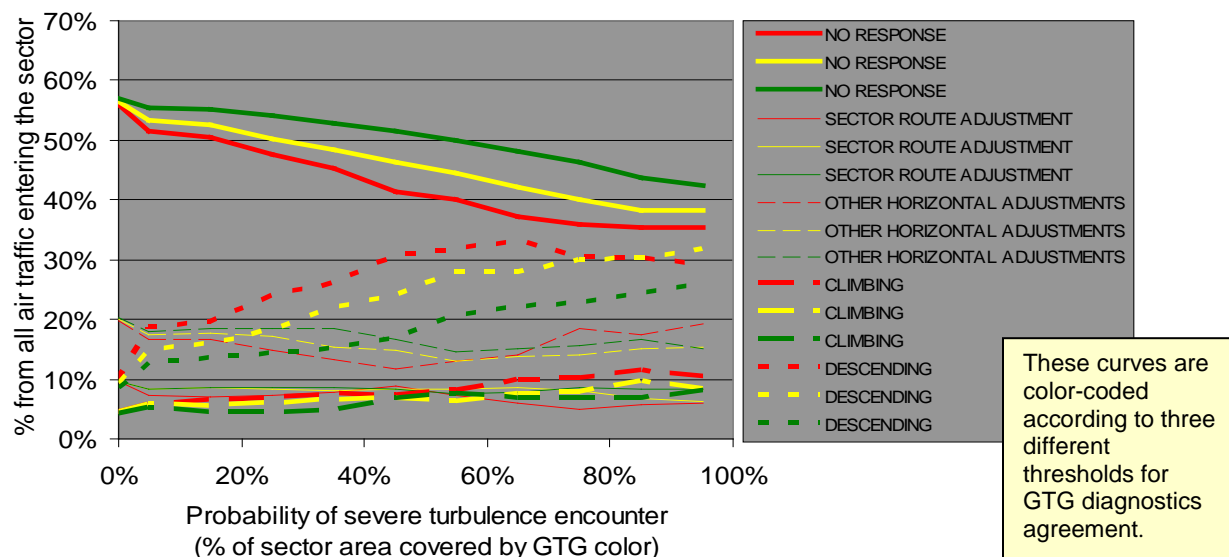


Figure 4: Distribution of turbulence-avoidance responses for air traffic entering an SFLP for a given probability of SoG turbulence encounter.

For the trajectory-based analysis, trajectories were analyzed for the presence of distinctive altitude and flight direction changes: that is, a constant altitude level or a straight flight path maintained for three or more track hits, followed by a continuous change to a new constant altitude or a straight flight path maintained for three or more 1-minute-spaced track messages. Likewise, only the first altitude or flight direction change was recorded as a response, even though a maneuver could consist of a series of two or more altitude or flight direction changes following one another. In any case, the percentage of multi-step responses within a 50-nmi stretch of flight was found to be negligibly small.

6. RESULTS OF THE ANALYSIS

We first present the results from the sector-based analysis, and then the trajectory-based analysis.

6.1 Sector-based Analysis

In the sector-based analysis, we analyzed sectors impacted by turbulence with a given relative probability of a SoG turbulence encounter for a given probabilistic GTG threshold, and determined the distribution of the pilot-behavior responses, as shown in Figure 4. Several important observations are made from these data.

Baseline Case: On a turbulence-free day (0% on the x-axis), 56% of all flights enter the next sector along their route without any adjustments; 10% amend their sector route; 8% adjust their altitude; 6% make some combination of altitude and sector route adjustment; and 20% resort to some other type of amendment (jet route, waypoints, etc.) while staying on the same sector route and at the same altitude. It is possible, but not confirmed, that these deviations may be due to underforecasts of MoG by GTG or to routine altitude adjustments when entering a new sector to avoid congestion for reasons not related to turbulence.

Turbulence Avoidance: As shown in Figure 4, the behavior of air traffic correlates with the relative probability of encountering SoG turbulence. The percentage of all air traffic that flies through a turbulent sector without any response decreases with the increasing likelihood of encountering SoG turbulence in that sector, while the percentage of aircraft that adjust their altitude by climbing or descending increases. The percentages of flights that only amend the sector route or something other than sector route and altitude stay roughly the same as the baseline case, suggesting that altitude adjustment (mainly descending) is indeed the favored response to encountered or anticipated turbulence.

For high likelihoods of encountering SoG turbulence, the percentages of flights that display various responses change relatively little. For example, for a high GTG probabilistic value above 80% turbulence coverage, about 35% of flights have no adjustments; 40% climb or descend; 6% engage in a sector route adjustment, and 19% resort to some other type of adjustment (jet route, waypoints, etc.).

Finally, we analyzed the distribution of climb and descent magnitudes as a function of the probability of a SoG turbulence encounter in the upcoming sector. Data in Figure 5 pertain to GTG SoG “probability” of ≥ 0.3 . As the probability of severe turbulence, as measured by sector coverage, becomes higher, the distribution for altitude maneuvers shifts towards descending and becomes wider. For the highest probabilities, the 4,000-ft descent predominates, while the likelihood of an 8,000-ft descent is almost the same as for a 2,000-ft climb. These data demonstrate that when there is a substantial probability of severe turbulence in a given SFLP, air traffic often avoids additional flight levels below and above. Given that gravity assists in a quick response, it is more likely that a pilot will quickly descend to an altitude clear of turbulence compared to climbing.

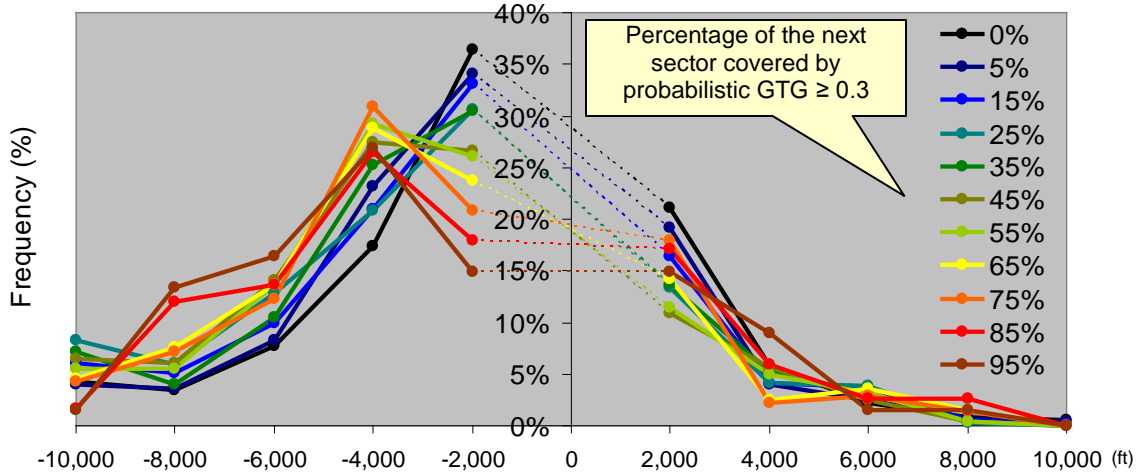


Figure 5: Magnitude distribution for sector-based altitude maneuvers for severe turbulence-avoidance for all aircraft types, based on sector-based turbulence probability.

6.2 Trajectory-based Analysis

In the trajectory-based analysis, we analyze different tactical responses to CAT and their dependence on various factors, such as an aircraft's user or weight class.

Figure 6 shows that for all aircraft attempting to avoid turbulence descending is the dominant pilot response to CAT. Since our analysis focused on turbulence within the next 50 nmi of travel, descent offers the quickest tactical solution. Climbing over CAT is an expected second choice, which may be favored by some aircraft (like GA) and strongly disfavored by others (like cargo carriers). Given that the turbulence boundary is only known probabilistically, and that aircraft are not equipped to look ahead to identify the exact turbulence boundary, we do not expect to see (nor did we observe) aircraft turning and re-routing to avoid turbulence. The statistics of turning to avoid turbulence always stays within the limits of statistical noise of the data, and, for that reason, is not shown in subsequent plots.

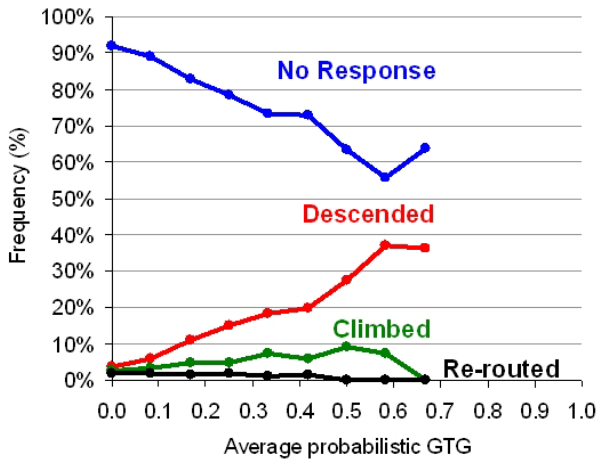


Figure 6: Maneuver statistics for all potential severe turbulence encounters for all aircraft analyzed.

Note that the GTG indices in Figure 6 are uncalibrated probabilistic values, and we have no precise way of evaluating how agreement among GTG diagnostics translates into actual probability of a severe turbulence encounter. This correlation is still an area of future research.

Next, we studied turbulence avoidance maneuvers categorized by user class. As categorized by TFMS, the user class includes: commercial, air taxi, general aviation, freight/cargo, military, and "other". Commercial refers specifically to commercial *passenger* aircraft.

Heavy passenger jets tend to respond to severe turbulence more pro-actively than heavy cargo jets (see Figure 7): given the same GTG probability of a severe turbulence encounter, a larger percentage of passenger jets will try to climb or descend out of the turbulent flight level. This is expected, as the possibility of turbulence-related injuries to passengers and/or passenger discomfort are key factors in commercial flight. This is confirmed by the statistics of response to moderate turbulence (Figure 7(a)). For moderate turbulence, passenger airline pilots are often concerned with passenger comfort, and avoid the turbulence more often than cargo airline pilots.

While we found that heavy commercial jets and large commercial jets respond very similarly, we observed that small GA jets respond more pro-actively than large GA jets, as shown in Figure 8. Some airlines exhibit more pro-active behavior than others, as shown in Figure 9. This may be explained by different airline policies established due to liability insurance and other airline preferences.

For the sector-based analysis, we further analyzed the trajectory-based data to identify the relationship between the magnitude of the altitude maneuvers and the probability of severe turbulence present along the upcoming flight path. As illustrated in Figure 10, higher probabilities of a severe turbulence encounter result in a) progressively larger dominance of descending over climbing, and b) progressively higher probabilities of altitude-changing maneuvers of large magnitude.

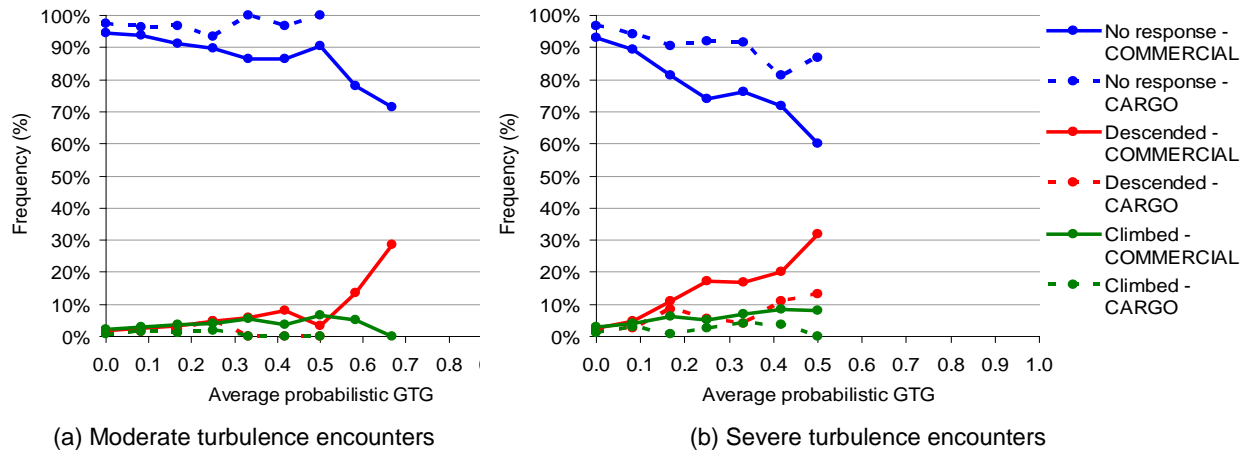


Figure 7: Comparison of turbulence avoidance maneuvers for heavy commercial and cargo jets encountering potential moderate and severe turbulence.

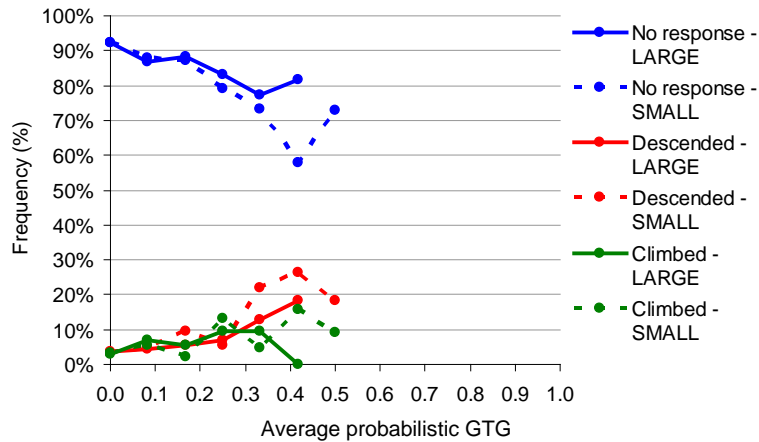


Figure 8: Comparison of turbulence avoidance maneuvers for small vs large GA jets encountering potential severe turbulence.

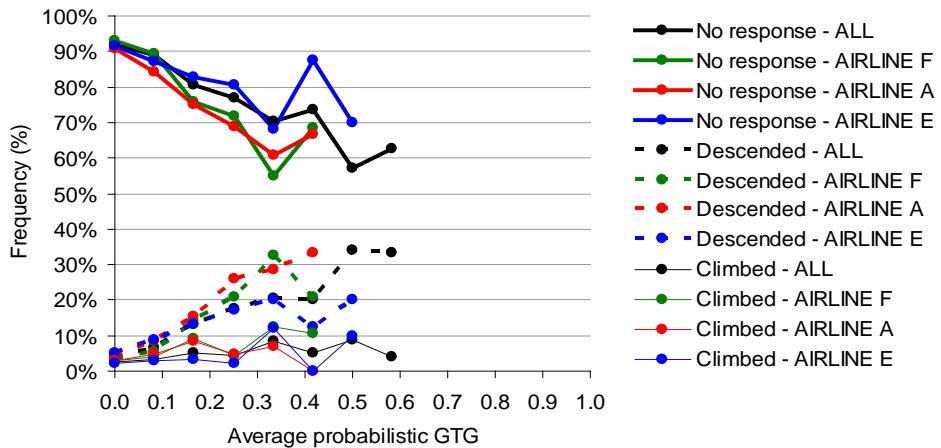


Figure 9: Comparison of turbulence avoidance maneuvers several commercial airlines (randomly labeled) encountering potential severe turbulence.

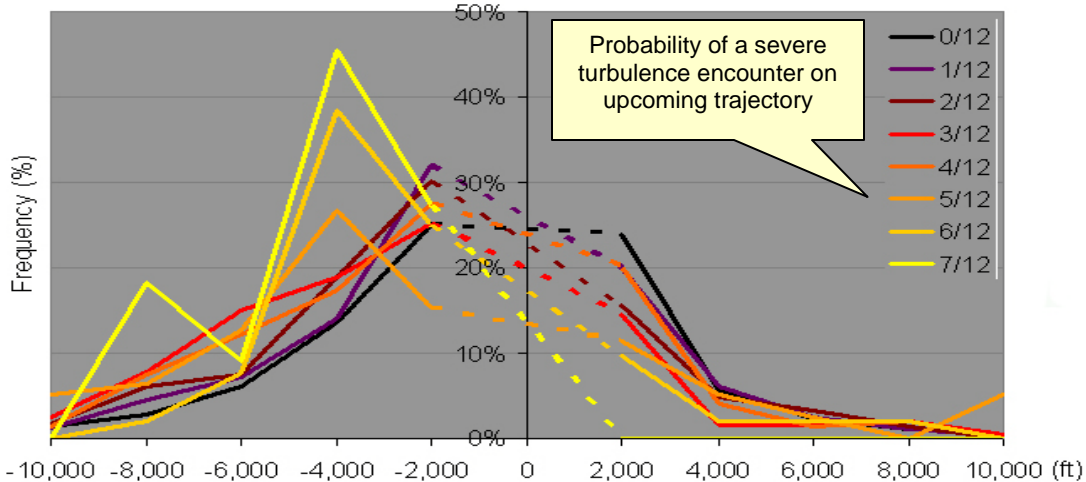


Figure 10: Magnitude distribution for trajectory-based altitude maneuvers for severe turbulence avoidance for all aircraft types, based on trajectory-based turbulence probability.

7. CONCLUSION

We have analyzed the impacts of Clear Air Turbulence (CAT) in the National Airspace System (NAS) using both a sector-based approach and a trajectory-based approach. CAT turbulence-avoidance maneuver statistics were classified by user class, weight class, physical class, aircraft type, as well as airline. Each of these factors plays a role in the maneuver chosen and magnitude of the CAT avoidance maneuver. General trends indicate that as the probability of severe-or-greater (SoG) turbulence increases for the upcoming sector or for the upcoming portion of flight trajectory, there is an increasing likelihood that the aircraft will maneuver, and the maneuver is most typically a vertical descent maneuver, increasing in magnitude as the probability of severe turbulence is higher. The analyses also show that some aircraft classes - for instance, cargo aircraft - are less likely to maneuver in moderate or severe turbulence, compared to passenger-carrying commercial aircraft; some airlines exhibit a more proactive policy than others; and small General Aviation (GA) jets respond more pro-actively than large GA jets.

For the Next Generation Air Transportation System (NextGen), to mathematically predict a response of air traffic given a turbulence probabilistic forecast, a series of models must exist to characterize the expected pilot behavior. In the future, such models must be maintained within decision support tools in order to characterize the sector airspace availability (sector-based analysis), flight-level by flight-level feedback of clear airspace and any potential density or workload limits in those airspaces (sector-based analysis), and expected user preferences given the expected level of turbulence (trajectory-based analysis).

ACKNOWLEDGEMENTS

This research was funded by NASA Ames Research Center under NRA contract NNA07BB36C for the NextGen Air Traffic Management (ATM) – Airspace

Project – Subtopic 15: Translation of Weather Information to Traffic Flow Management Impact, as well as under NRA contract NNA11AA17C, Weather Translation Models for Strategic Traffic Flow Management. The authors appreciate the frequent inputs from our contract monitor, William Chan.

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